Fundamental Wave Phenomena on Biased-Ferrite Planar Slab Waveguides in Connection With Singularity Theory

Alexander B. Yakovlev, *Senior Member, IEEE,* and George W. Hanson, *Senior Member, IEEE*

***Abstract*—In this paper, characteristic dispersion phenomena and interactions of the discrete surface- and leaky-wave modes supported by a grounded biased-ferrite slab waveguide are analyzed using singularity and critical-point theory. Surface- and space-wave leaky modes are studied for different orientations of the applied magnetic bias field. As the bias field is rotated away from a coordinate axis, the modes become hybrid, and mode coupling or modal degeneracies may occur. Mode coupling, in general, is governed by a Morse critical point (MCP), and the behavior of the MCP is found to be useful in explaining and predicting modal behavior on this complicated waveguiding structure. In addition, leaky-wave dispersion behavior and leaky-wave cutoff associated with a fold singular point is studied by varying the orientation of the applied magnetic bias field.**

***Index Terms*—Critical points, ferrite slab, leaky waves, mode coupling, singularity theory, surface waves.**

I. Introduction

FERRITE materials have been extensively used in various nonreciprocal devices such as phase shifters, polarizers, and isolators, where the electromagnetic properties of such devices can be controlled by varying the applied magnetic bias field [1]–[3]. A comprehensive literature review of microwave ferrite technology and various ferrite components, including circulators, isolators, phase shifters, YIG filters, and nonlinear magnetic microwave devices is given in [4].

Regarding fundamental modal phenomena, surface magnetostatic and dynamic modes supported by ferrite structures have been of primary interest for a long time. Surface magnetostatic and dynamic modes of a biased-ferrite slab waveguide were studied in [5]–[9], and magnetostatic volume and surface waves on printed strip and slot lines and circular microstrip resonators were recently investigated in [3], [10], and [11]. Complex modes in ferrite loaded parallel-plate waveguides and shielded finlines were analyzed in [12] and [13], respectively. There has also been some work on the analysis of leaky waves on ferrite slabs in the form of radiating space-wave modes [14], [15].

In this paper, we present a study of surface waves and space-wave leaky waves on a grounded ferrite slab waveguide biased with an arbitrarily oriented applied magnetic field (Fig. 1). The analysis is based on the numerical solution of an integral equation formulated for an equivalent volume current density [16] and uses principles of singularity theory. Of primary interest is the connection between various critical and singular points and modal phenomena. In this regard, Morse critical points (MCPs) were originally introduced in [17] and [18] for the analysis of modal coupling, and have been applied in a variety of guided-wave and resonant structures [19]–[25]. Singular points and complex-frequency branch points have been studied in [26] and [27] in the analysis of leaky-wave cutoff behavior on printed transmission lines. In this paper, fundamental modal phenomena are studied for biased-ferrite slabs with an arbitrarily oriented magnetic bias field (although numerical results are included only for the case of the bias field in the plane of the slab), with an emphasis on the application of critical-point theory.

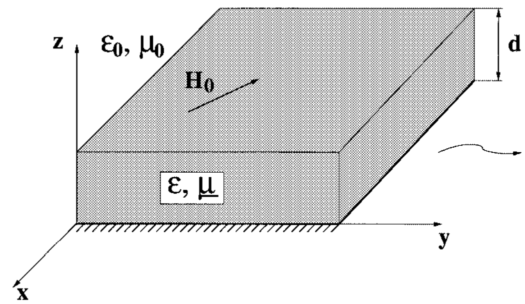

Fig. 1. Grounded ferrite slab waveguide with an arbitrarily oriented applied magnetic bias field.

II. Theory

Consider the two-dimensional planar waveguide depicted in Fig. 1, which is invariant along the waveguiding (y)-axis. A rigorous formulation of the natural-mode problem ($e^{j\omega t}$ time variation) yields

$$A\left(\lambda, \omega, \varepsilon, \underline{\mu}, d\right) X = 0 \tag{1}$$

where λ is the complex-valued modal propagation constant, ω is the angular frequency, ε is the dielectric permittivity of the ferrite slab, and X represents the amplitude coefficients in a modal-field distribution. The dyadic permeability $\underline{\mu}$ is given by

$$\underline{\mu} = \mu_0 R\left(\theta, \phi\right) \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} R^T\left(\theta, \phi\right) \tag{2}$$

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where $\mu = 1 + (\omega_0\omega_M)/(\omega_0^2 - \omega^2)$, $\kappa = \omega\omega_M/(\omega_0^2 - \omega^2)$, $\omega_0 = \gamma\mu_0 H_0$, $\omega_M = \gamma\mu_0 M_s$, H_0 is the dc magnetic bias field, M_s is the material saturation magnetization, $\gamma = -1.759 \times 10^{11}$ kg/coul, and

$$R(\theta,\phi) = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} \quad (3)$$

represents a rotation matrix that fixes the position of the bias field; R^T is the transpose of R.

Nontrivial solutions of (1) are obtained from the implicit dispersion equation

$$H\Big(\lambda, \omega, \underline{\varepsilon}, \underline{\mu}, d\Big) = \det\left(A\Big(\lambda, \omega, \underline{\varepsilon}, \underline{\mu}, d\Big)\right) = 0 \quad (4)$$

and, more generally, H is a mapping $\big(\lambda, \omega, \varepsilon, \underline{\mu}, d\big) \to C \in \mathbf{C}$, i.e., given ε, $\underline{\mu}$, and d only for certain values of (λ,ω) is $C = 0$.

By treating (λ,ω) as a pair of complex variables, a study of the properties of the mapping H leads to the analysis of critical points, singular points, and associated complex frequency-plane branch points, which explain modal phenomena [24]–[27]. Assuming that ε, $\underline{\mu}$, and d have specified values if

$$\begin{aligned} H'_\lambda\,(\lambda,\omega)|_{(\lambda_m,\omega_m)} &= 0 \\ H'_\omega\,(\lambda,\omega)|_{(\lambda_m,\omega_m)} &= 0 \\ \Delta(\lambda_m,\omega_m) &< 0 \end{aligned} \quad (5)$$

then $(\lambda,\omega) = (\lambda_m,\omega_m)$ is an MCP of the mapping H where

$$\Delta\,(\lambda_m,\omega_m) = H''_{\lambda\lambda}H''_{\omega\omega} - H''_{\lambda\omega}H''_{\omega\lambda}.$$

If

$$\begin{aligned} H\,(\lambda_f,\omega_f) = H'_\lambda\,(\lambda,\omega)|_{(\lambda_f,\omega_f)} &= 0 \\ H''_{\lambda\lambda}\,(\lambda,\omega)|_{(\lambda_f,\omega_f)}\; H'_\omega\,(\lambda,\omega)|_{(\lambda_f,\omega_f)} &\neq 0 \end{aligned} \quad (6)$$

then $(\lambda,\omega) = (\lambda_f,\omega_f)$ is a fold singular point of the mapping H. Both MCPs and fold singular points have been found to be useful in describing modal phenomena with fold singular points being associated with modal cutoff.

The inequality in (5) assumes a real-valued $\Delta(\lambda_m,\omega_m)$, which occurs for mode coupling on lossless waveguides. In the event of material loss, the first two conditions in (5) remain valid and define complex-valued critical points of the mapping H. Numerical experiments for lossy structures show that these critical points occur in the event of mode coupling, although the theoretical connection with coupled-mode theory is more difficult to establish. In this paper, we concentrate on lossless structures, such that the MCP (and $\Delta(\lambda_m,\omega_m)$) is real valued whenever mode coupling occurs.

III. Numerical Results and Discussions

Dispersion behavior of the first several modes on a biased-ferrite waveguide (Fig. 1) is shown in Fig. 2 for $d = 0.5$ cm, $\varepsilon = 15\varepsilon_0$, $H_0 = 100$ G, and $\mu_0 M_s = 1000$ G with the bias field oriented along the $(-x)$-axis ($\theta = 90°, \phi = 180°$). For each TE_n mode ($n > 1$), two fold singular points exist at the location of leaky-mode cutoff frequencies (one each for the forward and backward modes). Although the dispersion behavior is fairly complicated, there is no coupling between modes. Note that for this orientation of the bias field, TM modes are reciprocal (forward and backward waves propagate with the same velocity), although TE modes are significantly nonreciprocal. Dispersion curves for surface magnetostatic (the predominately vertical curves on the left-hand side of the plot) and dynamic modes shown in Fig. 2 are in excellent agreement with those presented in [8].

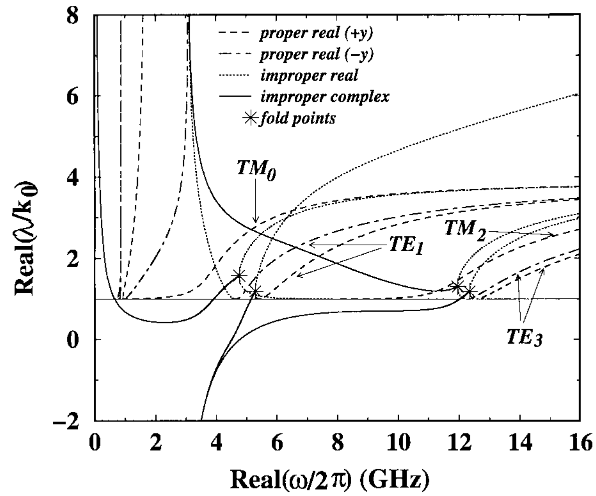

Fig. 2. Dispersion behavior of magnetostatic, TE, and TM surface waves and associated space-wave leaky waves on a grounded ferrite slab waveguide with $d = 0.5$ cm, $\epsilon = 15\epsilon_0$, $H_0 = 100$ G, and $\mu_0 M_s = 1000$ G. The applied magnetic bias field is along the $(-x)$-direction ($\theta = 90°, \phi = 180°$). The magnetostatic modes are the predominantly vertical curves emanating from approximately ($\omega/2\pi = 1$ GHz).

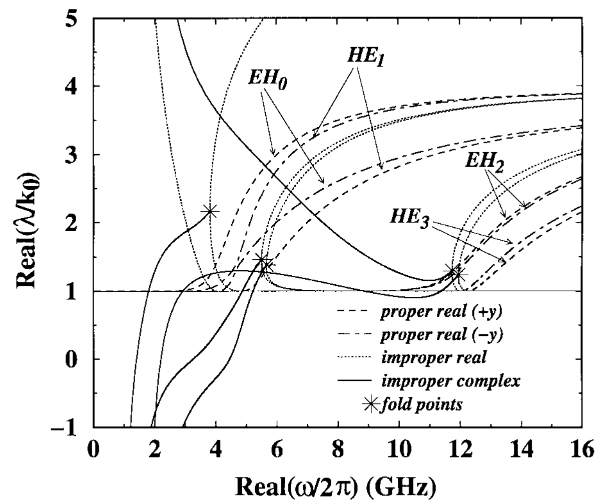

Fig. 3. Dispersion behavior of EH and HE hybrid surface waves and associated space-wave leaky waves on a grounded ferrite slab waveguide with the applied magnetic bias field oriented at $\theta = 90°$ and $\phi = 120°$. Mode coupling (mode transformation) of backward HE_1 and EH_0 surface waves occurs at approximately 4.7 GHz.

In Fig. 3, the magnetic bias field is rotated away from the x-coordinate axis ($\theta = 90°, \phi = 120°$). Dynamic modes are no longer purely TE or TM, and the dispersion behavior is more complicated than in the previous case. By rotating the magnetic bias field, TM modes become nonreciprocal (for instance, consider the EH_2 hybrid modes shown in Fig. 3), such that the

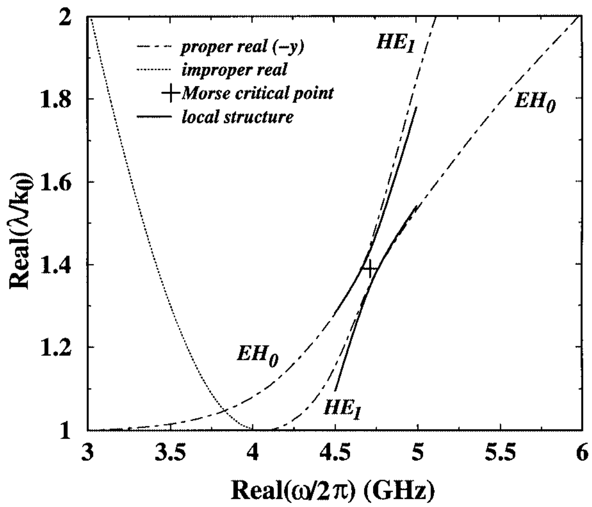

Fig. 4. Close-up of Fig. 3 ($\theta = 90°, \phi = 120°$) showing the mode transformation of backward HE_1 and EH_0 hybrid surface waves. An MCP is identified in the mode-coupling region.

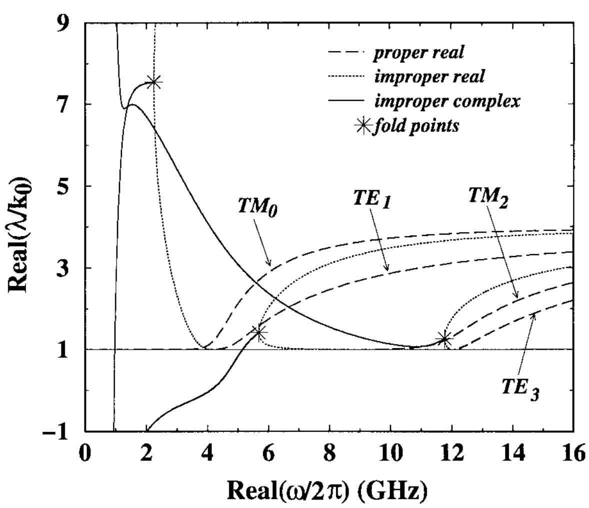

Fig. 5. Dispersion behavior of TE and TM surface waves and associated space-wave leaky waves on a grounded ferrite slab waveguide when the applied magnetic bias field is along the $(+y)$-direction ($\theta = 90°, \phi = 90°$). Note that forward and backward TE and TM surface waves propagate with the same velocity.

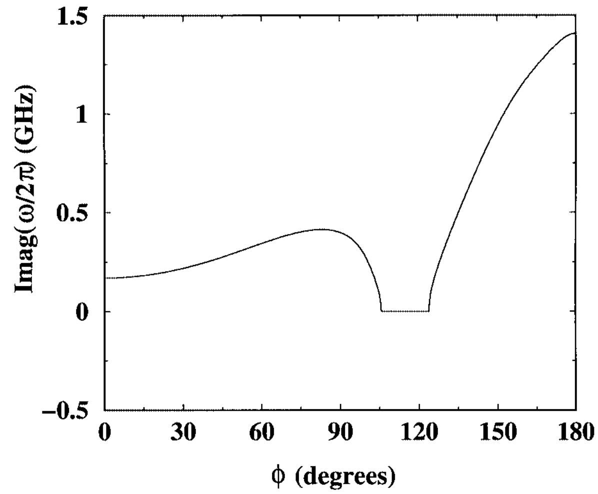

Fig. 6. Evolution of imaginary part of the complex Morse frequency with varying orientation of the magnetic bias field in the $(x-y)$-plane (ϕ-variation). It is found that the Morse frequency is only real valued within the range of $\phi \in [105.7°, 123.8°]$, which is associated with a mode-coupling region of backward HE_1 and EH_0 hybrid surface waves.

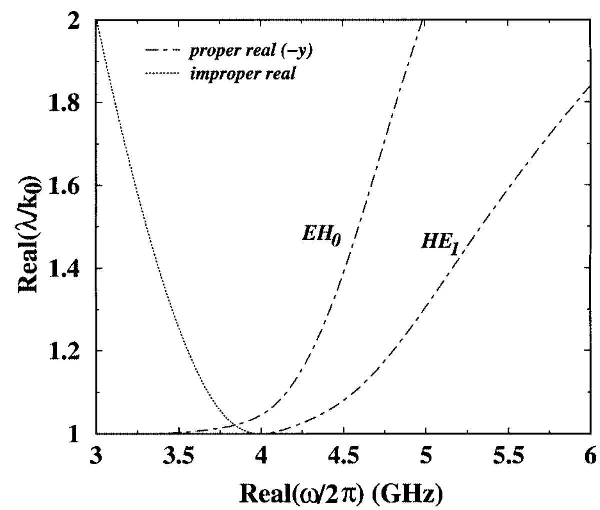

Fig. 7. Dispersion behavior of backward HE_1 and EH_0 hybrid surface waves when the applied magnetic bias field is oriented at $\theta = 90°$, $\phi = 100°$. Note that the modes do not couple (consistent with Fig. 6) and there is no mode transformation.

leaky-wave cutoff (fold point) in the reciprocal case splits into two cutoff points (two fold points) associated with forward and backward leaky waves. Also, mode coupling occurs between the hybrid HE_1 and EH_0 backward modes at approximately 4.7 GHz, resulting in characteristic hyperbolic-type behavior. Fig. 4 shows a close-up of the mode-coupling region where mode transformation $\mathrm{EH}_0 \leftrightarrow \mathrm{HE}_1$ occurs. Also shown in this figure is the associated MCP that governs this type of behavior, located at $(\lambda/k_0, \omega/2\pi) = (1.3906, 4.712)$. The local structure depicted in Fig. 4 represents qualitative and quantitative structural behavior in the mode-coupling region. It is obtained as a Taylor polynomial of order 2 of the dispersion function H in the vicinity of the MCP (e.g., see [24, eqs. (3) and (4)]).

As the bias field is further rotated to become oriented in the $(+y)$-direction ($\theta = 90°, \phi = 90°$), pure TE and TM modes again exist, and neither TE, nor TM modes exhibit nonreciprocal behavior, as shown in Fig. 5.

Since mode coupling is governed by the presence of an MCP [24], the behavior of the MCP can be used to predict the occurrence of mode coupling. For example, in Fig. 6, the imaginary part of the MCP is shown as the bias angle (ϕ) varies. Although the MCP exists for all $\phi \in [0,180°]$, as shown in this figure, it is only real valued for $\phi \in [105.7°, 123.8°]$. In this range, the MCP lies on the real-frequency axis, such that, as frequency varies, the MCP is encountered and mode coupling is exhibited. For $\phi \notin [105.7°, 123.8°]$, mode coupling does not occur since the MCP is not encountered as frequency varies (it lies off the real-frequency axis, in the complex plane) (see [25]). As an example, the EH_0 and HE_1 modes are shown in Fig. 7 for ($\theta = 90°, \phi = 100°$), where it is clear that the modes do not couple.

Mode coupling is generally considered to be the result of some perturbation of symmetry, in this case, due to a misaligned

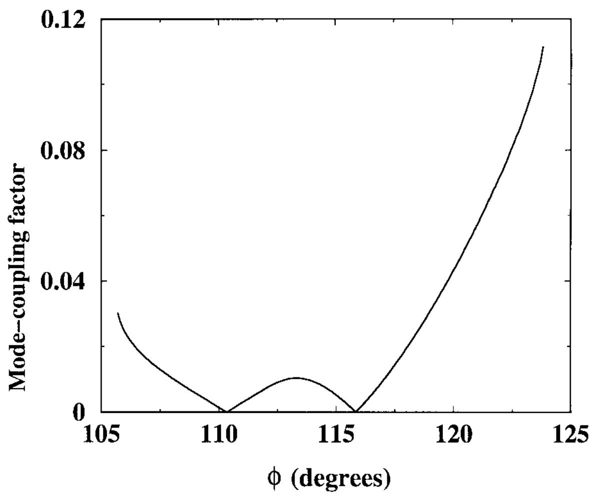

Fig. 8. Mode-coupling factor of backward HE_1 and EH_0 hybrid surface waves versus the orientation of the magnetic bias field in the $(x-y)$-plane (ϕ-variation). It is observed that at $\phi = 110.3°$ and $\phi = 115.8°$, the mode-coupling factor is zero, corresponding to the degeneracy of the hybrid waves.

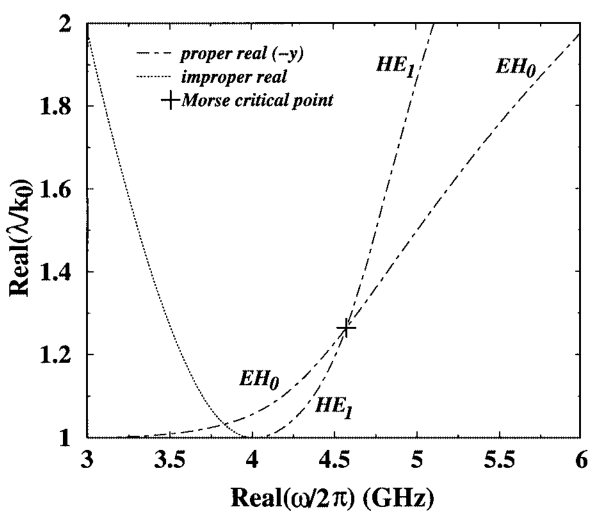

Fig. 9. Dispersion behavior of backward HE_1 and EH_0 hybrid surface waves when the applied magnetic bias field is oriented at $\theta = 90°$, $\phi = 115.8°$. A degenerate MCP is identified at the intersection of dispersion curves (a point of modal degeneracy).

bias field. As discussed above, even in the event of a misaligned bias field, mode coupling may exist only over a certain range of parameters, which can be explained by considering when the MCP lies on the real-frequency axis. However, within this range (e.g., $\phi \in [105.7°, 123.8°]$ for the structure discussed above), mode coupling may disappear at certain points that are associated with a modal degeneracy. The concept of an MCP can be used to fruitfully examine these phenomena. In particular, it is shown in [24] that, consistent with traditional coupled-mode theory, in general, coupled modes $\lambda_{1,2}$ can be related to uncoupled modes $\widetilde{\lambda}_{1,2}$ by

$$\lambda_{1,2} = \frac{\widetilde{\lambda}_1 + \widetilde{\lambda}_2}{2} \pm \sqrt{\left(\frac{\widetilde{\lambda}_1 + \widetilde{\lambda}_2}{2}\right)^2 \pm \left(\frac{K}{k_0}\right)^2} \qquad (7)$$

where the mode-coupling factor K/k_0 for codirectional coupled modes is

$$\frac{K}{k_0} = \sqrt{-2\frac{H(\lambda_m, \omega_m)}{H''_{\lambda\lambda}}} \qquad (8)$$

and where $H(\lambda_m, \omega_m)/H''_{\lambda\lambda} < 0$ for codirectional power flow and mode coupling between two forward (backward) traveling waves. If $K = 0$, by (7), the modes are degenerate ($\lambda_1 = \lambda_2$). For example, in Fig. 8, the mode-coupling factor is shown for the range of angles (ϕ) over which the MCP lies on the real-frequency axis. At two angles, $\phi = 110.3°$ and $\phi = 115.8°$, the mode-coupling factor is zero, such that the modes are degenerate. In Fig. 9, the EH_0 and HE_1 modes are shown for the degenerate case $\theta = 90°$, $\phi = 115.8°$. A degenerate MCP [having $H(\lambda_m, \omega_m) = 0$ in addition to the conditions (5)] is located at the point of modal degeneracy. Therefore, modal coupling is indicated by the presence of an MCP lying on the real-frequency axis, and modal degeneracies are indicated by the MCP being degenerate.

IV. Conclusion

Mode coupling and characteristic dispersion phenomena of surface waves and space-wave leaky waves on a grounded biased-ferrite slab waveguide have been analyzed using singularity and critical-point theory. Modal characteristics have been described for different orientations of the applied magnetic bias field. It has been shown that the behavior of the MCP can be used to characterize qualitatively and quantitatively the presence or absence of mode coupling and mode degeneracies.

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